From The Aeronautics File

By Max Feil

Topic #2: Center of Gravity & Pitch Stability

Introduction

Last time we talked about how a wing generates lift. This time I want to get more into airplane design, focusing on the topic of pitch stability or "where do I place that darn CG". If you've ever wondered what all those terms like center of lift, neutral point, aerodynamic center, and center of pressure really mean, then this is the article for you.

Before I get into the meat of the matter, I'll spend some time on a few very basic but important ideas. Most people know what a *force* is so I won't explain it here, but I will talk about its rotational equivalent. A *moment of force* (or *moment* for short) is simply a rotational force. One way to get a rotational force is by applying a normal (linear) force at some distance from a pivot point. The greater this distance or *moment arm* is, the larger the moment will be. When we talk about airplanes, we must realize that they are 3 dimensional objects moving through a 3 dimensional world. Such objects can move along 3 linear axes and 3 rotational axes, so we say that they have "6 degrees of freedom". They can move forward/backward, left/right, or up/down, and they can also pitch, yaw, and roll. We call the first three types of motion *translation*, and the last three types of motion *rotation*.

If we apply a linear force to an object such as an airplane, we can predict what it will do as long as we know where its center of gravity is. The center of gravity (CG), which is sometimes also called the center of mass, can be thought of as the pivot point of the object. Here is an example: Picture yourself as an astronaut floating in space. Your favourite model airplane is floating in front of you, motionless. You then use your finger to apply a force to different parts of the model. If the force you apply passes straight through the center of gravity, your model will move (translate) but it will not rotate. If the force you apply passes **almost** straight through the center of gravity, your model will **mostly** translate, but it will also rotate slightly. If the force you apply passes nowhere near the center of gravity, then your model will mostly rotate, and only slightly translate. As you can see, the farther from the center of gravity that you apply your force, the more of it gets turned into a rotational movement, and the less of it becomes a translational movement.

We can apply this idea to the effect of a single control surface on an airplane. If your airplane is flying along and you give up elevator, the immediate result will be determined by how big the force applied by the control surface is, and how far away it acts from the center of gravity. In the case of up elevator, the change in tail lift acts far away from the center of gravity producing a relatively large moment, so the result is mostly rotation (nose pitches up) and very little translation (loss of lift). If we compare this to application of wing flaps, we can see that in this second case the change in wing lift is acting much closer to the CG, so we have more translation (increase in lift) and less rotation. In both of the examples above (change in tail lift, change in wing lift) I did not mention that the immediate forces are followed by subsequent forces caused by changes in angle of attack. I will come back to this idea later since it is at the heart of pitch stability.

The Forces on Your Plane

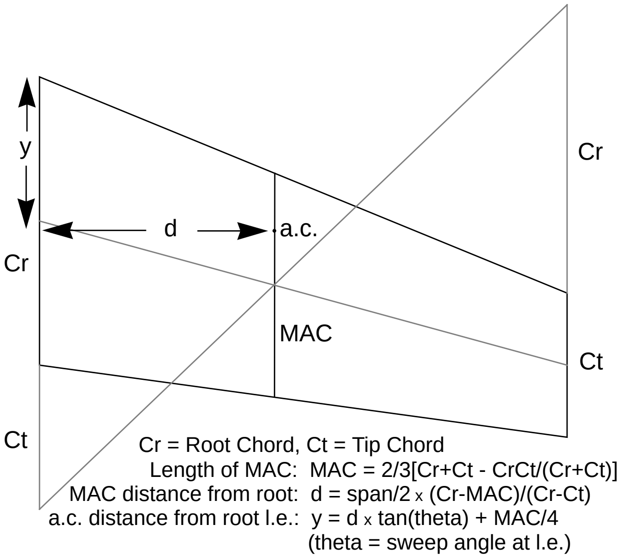
Now is a good time to talk about the forces applied to the different parts of your plane by the air. The important parts of the plane are the wing, the horizontal stabilizer, and the vertical fin. These three parts all work in very similar ways, and in fact all your plane really consists of is several wings held in position by a fuselage. How your plane acts is determined by the orientation of these "wings" (horizontal or vertical), their size and shape, and how far away they are from the center of gravity.

If we take a cross-section of one of these "wings" we get an *airfoil*, which is a simple 2 dimensional shape that can be used to describe many aspects of flight. The pressure distribution around the airfoil can be drawn as a bunch of tiny arrows showing the individual forces acting on the airfoil. We can combine these into a single lift force positioned at what we call the *center of lift* or *aerodynamic center* of the airfoil and which acts perpendicular to the oncoming air. Along with this translational force there is almost always a rotational moment about the aerodynamic center since the individual tiny forces don't all point in the same direction. Note that we could have chosen any point on the airfoil and talked about the lift produced there and the rotational moment around it, however we chose the aerodynamic center since it has one very important property: The magnitude of the aerodynamic moment around it does not change with angle of attack, only the magnitude of the lift vector does. This makes it a very convenient spot, worthy of the name "aerodynamic center". Now for a very astounding fact: The aerodynamic center of ALL airfoils is at almost exactly 25% of the way along the chord from the leading edge. This does not change - ever. The only exceptions are when the air flow separates from the airfoil, or when the speed of flight approaches mach 1. In supersonic flight, the aerodynamic center of all airfoils moves back to almost exactly 50% of their chord, but very few of us R/C modelers will ever encounter this kind of speed with our models!

We can extend the idea of a 2 dimensional airfoil (infinite span wing) to a three dimensional (finite span) wing. The aerodynamic center or center of lift of a wing has all the same properties as was described for an airfoil, except it is harder to talk about the chord of a wing, especially if the wing is swept and/or tapered. In such cases we use a special mean (or average) aerodynamically equivalent chord in all of our calculations. This *mean aerodynamic chord*, or MAC for short, can usually be derived with simple high school geometry and formulas (see figure 1). A more precise definition of the MAC is "the chord through the centroid of your wing's plan area, whose aerodynamic center is representative of the wing as a whole." If your wing has a simple rectangular planform, like that of many R/C aircraft, the MAC is really easy to figure out - it's the same as the chord half way along the span. If your wing is swept and/or tapered (i.e. has a trapezoidal shape), the method shown in figure 1 can be used to find out the length and position of the MAC. For more complex wings, such as those with multiple tapers or curved outlines, first break up the wing into equal span trapezoids, then use the method in figure 1 on each trapezoid and combine the results using an area-weighted average. In any case, once you know where the MAC is on your airplane's wing, you can be certain that the aerodynamic center and the center of lift of the wing will be 25% back from the leading edge of the MAC.

Figure 1:

Geometrical method for MAC determination, using half of wing's span: Add the length of the tip chord below the root chord and the length of the root chord above the tip chord. Draw a line between the two endpoints just created. Where this line intersects the line from the midpoint of the root chord to the mid point of the tip chord is the centroid of the trapezoidal wing shape and indicates the location of the MAC. The aerodynamic center of the wing will be one quarter of the way along the MAC from its leading edge.



Cr = Root Chord, Ct = Tip Chord
Length of MAC: MAC = 2/3[Cr+Ct - CrCt/(Cr+Ct)]
MAC distance from root: d = span/2 x (Cr-MAC)/(Cr-Ct)
a.c. distance from root l.e.: y = d x tan(theta) + MAC/4
(theta = sweep angle at l.e.)

You may have guessed by now that we can extend the idea of aerodynamic center from the “wings” on an airplane (i.e. main wing and stab) to the entire airplane. The aerodynamic center or center of lift of the entire airplane will be somewhere between the centers of lift of the wing and stab, and closer to the larger, more effective lift producer. When you think about it, this makes perfect sense. What may not be so obvious is that this point on your airplane is fixed and never moves, as long as your airplane keeps flying and as long as it doesn’t break the sound barrier! Angle of attack, airfoil selection, and angles of incidence do not affect the position of the plane’s aerodynamic center. In fact, if you take two totally different airplanes and they have the same relative sizes and shapes of wing and tail, their aerodynamic centers will be in the same relative position. This is something to keep in mind when making rough sketches of a new design, since starting with a proven outline (top view) will help ensure your chance of success.

The airplane’s aerodynamic center or center of lift is also called the *Neutral Point* (NP) due to its effect on stability of the aircraft, which I will talk about in a minute. First I’ll take a second to recap the last few paragraphs. We talked about airfoils, then wings made up of airfoils, then airplanes made up of wings. Each of these objects has one special point called the *aerodynamic center* or *center of lift* or *neutral point*. All three of these names refer to the same spot, but we must always be careful to mention whether we are referring to a 2-d airfoil, a wing, or to the entire airplane. In normal usage, when we say “center of lift” we are usually talking about a wing, and when we say “neutral point” we are usually talking about the entire aircraft, but this may not always be the case. As an example consider a flying wing aircraft. The wing center of lift and aircraft neutral point will be at the same spot since there is no tail!

Center of Pressure Debunked

Now is probably the best time to mention the *center of pressure*, which is likely the most misunderstood concept in “layman aerodynamics”. Luckily it is also quite meaningless when it comes to aircraft design. Remember when I introduced the center of lift earlier, I also said that you cannot talk about the translational lift acting at that point without always remembering the rotational moment about that point. Well the center of pressure is a tricky combination of both of these ideas (translation and rotation), and as a result becomes intuitively hard to deal with. One way to define center of pressure (CP) is “that point at which the lift vector would have to act in order to create the correct rotational moment about the aerodynamic center.” As an example, consider a normal flat bottom wing. Its center of lift will be at 25% MAC. Wings of this type (positive camber) all have an inherent negative (leading edge down) pitching moment, which is constant about the aerodynamic center for all angles of attack before stall. But as the angle of attack of a wing changes, the amount of lift it produces also changes. Because of the way it is defined, the wing’s CP must move as the size of the lift vector increases or decreases in order to keep the negative pitching moment of the wing constant. As the angle of attack and therefore the lift of a wing increases (such as at the start of a loop), the center of pressure moves forward from a position behind the aerodynamic center, getting close to but never quite reaching the aerodynamic center. Similarly, when angle of attack decreases (such as when you push down on the stick to lose some altitude or do an outside loop), the wing’s CP moves backward. When the lift vector reaches zero (at an angle of attack just under 0 degrees) the CP will actually be millions of miles behind the wing, at minus infinity! As angle of attack decreases further, the CP pops over to plus infinity and starts approaching the aerodynamic center from the front. All this movement of the CP happens without any change in the pitching moment of the wing, which is constant. In other words, a change in the position of the CP is totally meaningless when trying to make conclusions about rotational motion or forces. For this reason I will not use it at all when talking about pitch stability or center of gravity location. Anybody I have ever heard try to do so has either made a total mess, or has used the term center of pressure by mistake when they really should have said aerodynamic center or neutral point. One final point about center of pressure: Symmetrical airfoils do not have a built-in pitching moment, therefore the center of pressure does not move with changes in the magnitude of the lift vector. The wing’s CP will always be right at the aerodynamic center, unless the wing’s camber changes through the use of flaps or ailerons. As an aside, since model rockets are almost always symmetrical objects with no control surfaces, model rocket designers use the term CP instead of NP when talking about rocket stability, perhaps without even realizing that they would not be able to do so in different circumstances.

Pitch Stability

We have finally arrived at the point where we can talk about stability. But first, what do I mean by stability? Stability is simply a tendency to go against a disturbance. For example, if your plane is flying along and a gust of wind causes its nose to pitch up, you want your plane to immediately oppose that motion so that the pitch up is limited. This type of stability is called static longitudinal stability. If your plane was statically unstable, then a slight pitch up caused by some disturbance would lead to even more pitch up, which in turn would cause even more pitch up, and so on. The result in an extreme case would be a very fast transition from level flight to a vertical attitude with possible shedding of wings or complete loss of pitch (elevator) control. In a mild case of instability or even reduced stability, the model will seem “squirrely” in pitch, with the pilot having to make frequent corrections for unexpected pitch changes. Some airplanes are designed with relaxed pitch stability, but in those cases a computer is often used to quickly respond to pitch changes and make corrections on behalf of the pilot to keep the plane flying straight.

I will now tell you the cardinal rule of pitch stability: For an airplane to be stable in pitch, its center of gravity must be forward of the airplane’s neutral point by a safety factor called the static margin. The static margin is usually expressed as a percentage of the length of the MAC. If the static margin is zero (i.e. CG right at NP) then the aircraft is termed “neutrally stable”, hence the term “neutral point”. For conventional airplane designs (i.e. planes with a wing and a stab) the static margin should be between 5% and 15% of the MAC. An example will help here. Consider a typical high wing trainer. A very likely NP location for this aircraft would be at 38% of the MAC of its main wing. This can vary quite a bit depending on the size of the wing vs. the tail and the distance between the wing and the tail, but 38% MAC is a perfectly realistic number. If we wanted to use a 10% MAC static margin, we would come up with a desired CG location of 28% MAC. Note that this location is just slightly behind the center of lift of the main wing, which is at 25% MAC. This means that our tail will be slightly lifting in straight and level flight, unless the negative pitching moment of the wing is big enough to overcome the entire moment that is produced by having the CG behind the CL of the main wing. Whether the tail lifts or not has no effect on stability, as long as we have an appropriate static margin. In fact since the most common rule of thumb for R/C planes is to put the CG 30% back from the leading edge of the MAC, and since this is 5% behind the main wing's center of lift, I'll bet that a lot more R/C planes fly with lifting tails than most people think.

Now let's see why having the CG in front of the NP makes an airplane stable. Remember that the NP is the center of lift of the entire aircraft. If a disturbance causes the plane to pitch up, the angle of attack of both the main wing and the stabilizer will increase, which causes an increase in the magnitude of the lift vector at the airplane's NP but no change in the rotational moment about the NP. We now have the equivalent of a linear force acting behind the center of gravity. As was discussed at the beginning of this article, this will cause a nose-down moment which will oppose the original disturbance. Simple, isn't it? A similar sequence of events takes place if the disturbance is in the opposite direction, except the restoring moment will be nose-up. This explanation can also be used to illustrate the unstable condition when the CG is behind the NP. In that case the extra linear lift force caused by the disturbance will create a moment in the same direction as the disturbance. This starts a very quick "vicious cycle" that will flip the plane out of control before a pilot has time to react. See figure 2 for a diagram which shows the stability-related forces acting on airplanes of several different configurations.

<u>Figure 2:</u>
Force setups of various airplane configurations. There are five general CG locations, depending on your model's design. They are:

2.1 Conventional. CG at 20 to 25% of MAC from the MAC leading edge, or a bit further back, as far as 28% or more (depending on the airfoil) to overcome part or all of the main wing's negative pitching moment.

2.2 Conventional with lifting tail. CG located as far aft as 80% of the MAC from the MAC leading edge.

2.3 Tandem wings (fore and aft wings of roughly equal areas). CG located somewhere between the wings.

2.4 Canard (horizontal tail first). CG located ahead of aft larger wing.

2.5 Flying wing. CG at 9 to 17% of MAC.

Fig. 2-1. Force arrangement for longitudinal stability in a conventional, rear-tailed aircraft. The CG acting ahead of the CL combines with the (normally) negative pitching moment if a cambered wing section is employed. The resulting nose down force must be overcome by a "down load" on the horizontal tailplane.:

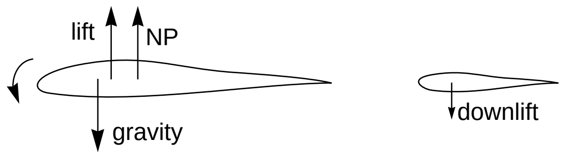


Fig. 2-2. Force arrangement for longitudinal stability in a lifting tail (eg. free-flight) model. As always the NP is behind the CG. The tailplane is normally an airfoil section, usually flat bottomed and contributes to the overall lift, offsetting the effect of the aft CG. This force setup is used on high performance free flight models, with horizontal tail surface areas roughly 35 to 45% of their wing areas, which moves the NP rearward:

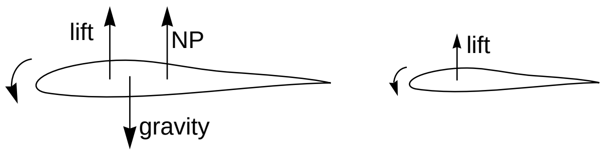


Fig. 2-3. Force arrangement for longitudinal stability in a tandem-winged aircraft. The NP is roughly halfway between the 1/4 chord points of the two wings, with the CG ahead of it:

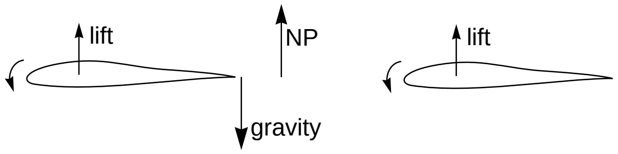


Fig. 2-4. Force arrangement for longitudinal stability in a canard aircraft. The NP is further back, closer to the larger aft wing, and the CG is ahead of it:

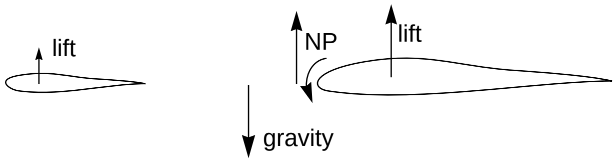
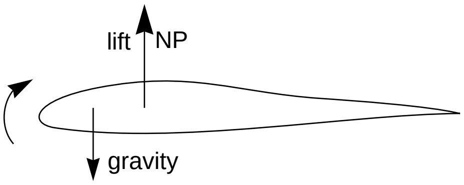


Fig. 2-5. Force arrangement for longitudinal stability in a flying wing. NP is right at wing CL. Wing must have a POSITIVE pitching moment to balance the CG in front of the CL. This is usually provided by a reflexed airfoil, and/or sweep plus washout

and/or raised elevons, etc.:



Designing Your Own Airplane

When you design your own plane, whether its configuration is conventional, lifting tail, tandem wing, canard, or flying wing, you now know the two main design parameters for giving it adequate longitudinal stability: The Neutral Point location, and the static margin. You must determine these before you can tell where the CG should be.

Determining the NP location: The most basic way of locating the NP is to use the relative areas of the two horizontal lifting surfaces, and locate the NP proportionately along the line between the 25% MAC chord points of the two wings. In other words, percentage distance = (wing1 area)/(wing1 area + wing2 area). For example a tandem wing aircraft with wings of equal area will (according to this simple formula) have an NP that is about mid way between the two quarter-chord points of the two wings. Similarly, if an airplane has a canard that is 50% the size of the main (aft) wing, then this formula gives an NP location that is one-third of the way between the main wing's CL and the canard's CL.

There are complicating factors, however, that make the simple area based formula inaccurate. If the two "wings" have different aspect ratios, then the NP will be located closer to the one that is slimmer (and therefore more efficient). Also, since the rear wing operates in disturbed air, the NP will be more forward than the simple calculation predicts. The fuselage and propeller affect the location of the NP as well, as does the presence of floats, etc. In spite of all this, if you understand at least the major factors that affect the NP location, you can add a "fudge factor" and arrive at a much better starting point for flight testing. If you are interested in more complex methods for NP determination, I can direct you to several sources for this kind of information.

Determining the static margin: A good rule of thumb for conventional, lifting tail, and flying wing designs is to make the static margin 5 to 15 percent of the MAC of the main wing. For tandem wing and canard designs, 15 to 25% or more may be needed, especially if your NP calculation is too simple. Using a conservative CG location as a starting point, you can begin flight testing. Keep in mind that there are ways of reducing the risk of flight testing. The March 1993 issue of Radio Control Modeler (RCM) magazine had a really good article on doing initial flight testing using small balsa "chuck gliders" of your intended design. The article was entitled "Model Modeling - Pre-testing your design", by Clay Ramskill.

Flying Wing Example

I will now give an example of pitch stability calculations for a non-conventional type of airplane. I own a Klingberg Wing flying wing glider and a few months ago I wanted to calculate where I would put the CG so I could compare this to where the plans show the correct CG range. The first thing I had to figure out was the location of the neutral point of the aircraft. Luckily on a flying wing this is easy since it will be the same as the wing center of lift - at 25% of the MAC. The Klingberg Wing is swept and tapered, so I used the geometrical method shown in figure 1 to determine the mean aerodynamic chord. I ran into difficulty with the next step since at the time I had no idea how big the static margin should be. At this point I decided to look at the plans and calculate backwards. According to my numbers, they showed the allowable CG range to be between 9% and 17% MAC. This means that the static margin they recommend is between 8% and 16% MAC. This is not too different than the 5% to 15% recommendation for conventional designs. I can attest to the need for at least 8% static margin on the Klingberg wing, since at first I had the CG very close to the rearward limit - maybe even a little behind it. During the first few test flights I lost control several times in tight turns, resulting in spiral dives into the ground. This taught me to respect the rearward CG limit shown on plans, especially for unconventional designs. To fix the stability problem, I moved the CG forward to increase the static margin from 7% to about 12% MAC.

The Final Step

Now that we know how to figure out the approximate CG position needed to make our airplane stable in pitch, the final step is to choose an exact spot for the CG within the range of allowable positions. As we move rearward within this range, the airplane will become slightly more unstable but also more responsive in pitch. As we move forward, the airplane will become more stable but we may run out of elevator authority, especially when flaring for landings.

The spot you choose for your CG within the stable range is to some extent up to your personal preference - how you want your airplane to handle. However sometimes there are additional factors involved such as improving ground handling, or trying to increase efficiency by reducing drag. Let's look at a conventional design: a high performance glider. We can gain efficiency by making the tail not provide any lift in cruising flight - remember that lift also means drag, and that the wing is more efficient at providing lift than the tail. To do this we simply place the CG at the aerodynamic "balance point" of the main wing. For symmetrical airfoils, this will be at exactly the center of lift, or 25% MAC. For most other airfoils, this point is a little behind the center of lift since we have to take into account the negative pitching moment of the wing itself. An example CG location might then be 28% MAC. Now we have a tail that does not normally provide any down lift or up lift. Our glider is still stable since we still have a healthy static margin (NP will be at 35% MAC or more). The glider, when perturbed by a gust of wind, will have pitch stability since the tail will momentarily provide up or down lift to help straighten out the nose.

Summary

That's it for another installment of "From the aeronautics file". I hope that what I said made some sense. The main points to remember are:

- Airfoils, wings, and airplanes all have a very important point referred to by one of three names: aerodynamic center, center of lift, neutral point.
- The "center of pressure" is a strange beastie that is useless for our purposes.
- For adequate longitudinal (i.e. pitch) stability, the CG must be forward of the NP by a safety factor called the "static margin".
- The NP location depends mostly on the relative areas and locations of the forward vs. the rear "wings" on an airplane, and will be located in proportion to those areas (with a "fudge factor") somewhere between the quarter chord points of those wings.

Next time I will explain why dihedral provides roll stability. The answer may surprise you!